MAC 2311
Chapter 3 Review Materials (Part 2)
Topics Include The Mean Value Theorem, Optimization, Newton's Method, and Antiderivatives

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the value or values of c that satisfy the equation $\frac{f(b) - f(a)}{b - a} = f'(c)$ in the conclusion of the Mean Value Theorem for the function and interval.

1) $f(x) = x^2 + 5x + 4$, $[2, 3]$ 1) ________

A) 2, 3
B) $0, \frac{5}{2}$
C) $\frac{5}{2}$
D) $-\frac{5}{2}, \frac{5}{2}$

Determine whether the function satisfies the hypotheses of the Mean Value Theorem for the given interval.

2) $f(x) = x^{1/3}$, $[-5,4]$ 2) ________

A) Yes
B) No

3) $s(t) = \sqrt{t(5 - t)}$, $[-1,5]$ 3) ________

A) No
B) Yes

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Answer the question.

4) A trucker handed in a ticket at a toll booth showing that in 3 hours he had covered 225 miles on a toll road with speed limit 65 mph. The trucker was cited for speeding. Why? 4) ________________

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

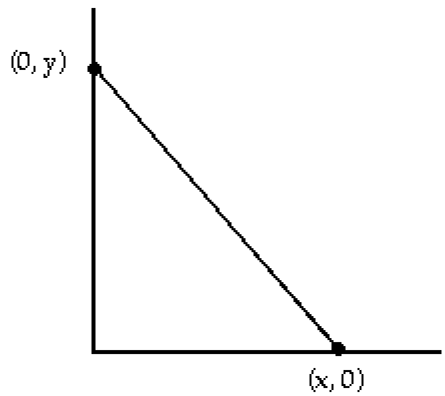
Solve the problem.

5) From a thin piece of cardboard 30 in. by 30 in., square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume? Round to the nearest tenth, if necessary. 5) ________

A) 20 in. × 20 in. × 5 in.; 2000 in^3
B) 10 in. × 10 in. × 10 in.; 1000 in^3
C) 15 in. × 15 in. × 7.5 in.; 1687.5 in^3
D) 20 in. × 20 in. × 10 in.; 4000 in^3

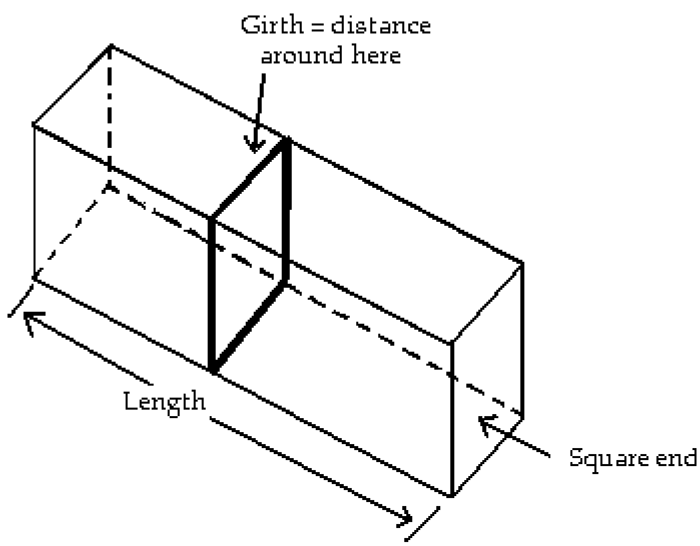
SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

6) You are planning to close off a corner of the first quadrant with a line segment 21 units long running from (x, 0) to (0, y). Show that the area of the triangle enclosed by the segment is largest when x = y. 6) ________________



MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

7) A private shipping company will accept a box for domestic shipment only if the sum of its length and girth (distance around) does not exceed 120 in. What dimensions will give a box with a square end the largest possible volume? 7) ________

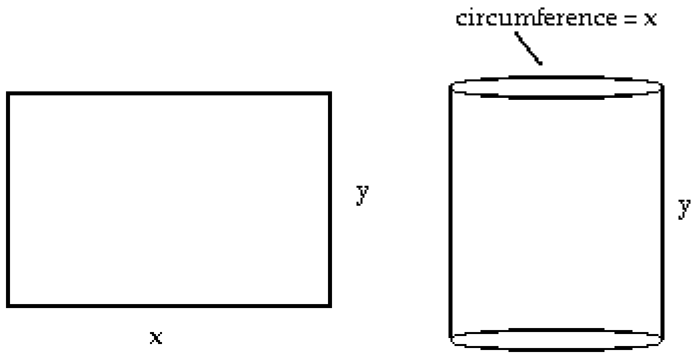


A) 20 in. × 20 in. × 40 in.
B) 20 in. × 40 in. × 40 in.
C) 20 in. × 20 in. × 100 in.
D) 40 in. × 40 in. × 40 in.

8) Determine the dimensions of the rectangle of largest area that can be inscribed in a semicircle of radius 3. 8) ________

A) $h = 3\sqrt{2}$, $w = \frac{3\sqrt{2}}{2}$,
B) $h = 3\sqrt{2}$, $w = \sqrt{2}$
C) $h = \frac{3\sqrt{2}}{2}$, $w = 3\sqrt{2}$
D) $h = \sqrt{2}$, $w = 3\sqrt{2}$

9) A rectangular sheet of perimeter 27 cm and dimensions x cm by y cm is to be rolled into a cylinder as shown in part (a) of the figure. What values of x and y give the largest volume? 9) ________

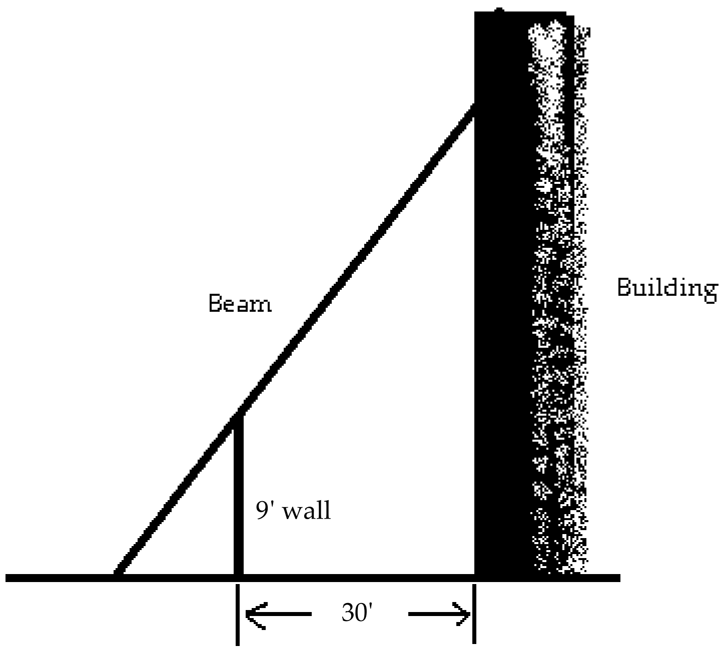


A) x = 11 cm; y = $\frac{5}{2}$ cm
B) x = 8 cm; y = $\frac{11}{2}$ cm
C) x = 10 cm; y = $\frac{7}{2}$ cm
D) x = 9 cm; y = $\frac{9}{2}$ cm

10) A private shipping company will accept a box for domestic shipment only if the sum of its length and girth (distance around) does not exceed 120 in. Suppose you want to mail a box with square sides so that its dimensions are h by h by w and it's girth is 2h + 2w. What dimensions will give the box its largest volume? 10) ________

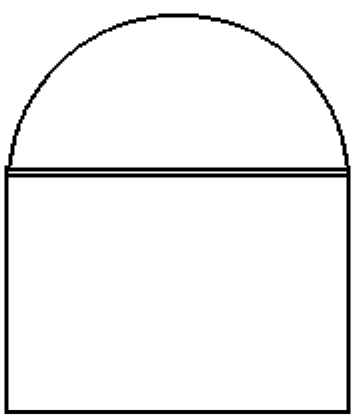
A) 20 in. × 20 in. × 100 in.
B) 20 in. × 20 in. × 40 in.
C) $\frac{80}{3}$ in. × $\frac{80}{3}$ in. × 20 in.
D) 40 in. × 20 in. × 40 in.

11) The 9 ft wall shown here stands 30 feet from the building. Find the length of the shortest straight beam that will reach to the side of the building from the ground outside the wall. 11) _______



A) 53.3 ft B) 52.3 ft C) 51.3 ft D) 39 ft

12) A window is in the form of a rectangle surmounted by a semicircle. The rectangle is of clear glass, whereas the semicircle is of tinted glass that transmits only one–fifth as much light per unit area as clear glass does. The total perimeter is fixed. Find the proportions of the window that will admit the most light. Neglect the thickness of the frame. 12) _______



A) $\frac{\text{width}}{\text{height}} = \frac{5}{10 + 4\pi}$ B) $\frac{\text{width}}{\text{height}} = \frac{20}{10 + 4\pi}$

C) $\frac{\text{width}}{\text{height}} = \frac{20}{5 + 4\pi}$ D) $\frac{\text{width}}{\text{height}} = \frac{20}{10 + \pi}$

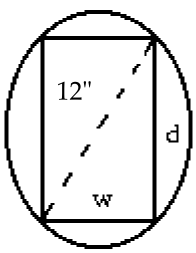
13) At noon, ship A was 15 nautical miles due north of ship B. Ship A was sailing south at 15 knots (nautical miles per hour; a nautical mile is 2000 yards) and continued to do so all day. Ship B was sailing east at 8 knots and continued to do so all day. The visibility was 5 nautical miles. Did the ships ever sight each other? 13) ________

A) Yes. They were within 4 nautical miles of each other.
B) No. The closest they ever got to each other was 7.1 nautical miles.
C) Yes. They were within 3 nautical miles of each other.
D) No. The closest they ever got to each other was 8.1 nautical miles.

14) A small frictionless cart, attached to the wall by a spring, is pulled 10 cm back from its rest position and released at time t = 0 to roll back and forth for 4 sec. Its position at time t is $s = 1 - 10 \cos \pi t$. What is the cart's maximum speed? When is the cart moving that fast? What is the magnitude of of the acceleration then? 14) ________

A) $10\pi \approx 31.42$ cm/sec; t = 0.5 sec, 1.5 sec, 2.5 sec, 3.5 sec; acceleration is 0 cm/sec^2
B) $\pi \approx 3.14$ cm/sec; t = 0.5 sec, 1.5 sec, 2.5 sec, 3.5 sec; acceleration is 0 cm/sec^2
C) $10\pi \approx 31.42$ cm/sec; t = 0.5 sec, 2.5 sec; acceleration is 1 cm/sec^2
D) $10\pi \approx 31.42$ cm/sec; t = 0 sec, 1 sec, 2 sec, 3 sec; acceleration is 0 cm/sec^2

15) The stiffness of a rectangular beam is proportional to its width times the cube of its depth. Find the dimensions of the stiffest beam than can be cut from a 12-in.-diameter cylindrical log. (Round answers to the nearest tenth.) 15) ________



A) w = 7.0 in.; d = 9.4 in.
B) w = 6.0 in.; d = 10.4 in.
C) w = 5.0 in.; d = 11.4 in.
D) w = 7.0 in.; d = 11.4 in.

Use Newton's method to estimate the requested solution of the equation. Start with given value of x_0 and then give x_2 as the estimated solution.

16) $3x^2 + 2x - 1 = 0$; $x_0 = 1$; Find the right-hand solution. 16) ________

A) 0.35 B) 0.33 C) 0.50 D) 0.85

17) $-x^2 + 4x - 1 = 0$; $x_0 = 0$; Find the left-hand solution. 17) ________

A) 0.14 B) 0.25 C) 0.23 D) −0.33

18) $x^3 + 5x + 2 = 0$; $x_0 = -1$; Find the one real solution. 18) ________

A) −0.44 B) −0.38 C) −0.39 D) −0.64

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Solve the problem.

19) Use Newton's method to estimate the solutions of the equation $4x^2 + 3x - 2 = 0$. Start with $x_1 = 0.5$ for the right–hand solution and with $x_0 = -2$ for the solution on the left. Then, in each case find x_2. 19) ______________

20) Use Newton's method to estimate the one real solution of $-2x^3 - 3x - 4 = 0$. Start with $x_1 = -0.5$and then find x_2. 20) ______________

21) Use Newton's method to estimate the solution of the equation $2\sin x - 3x + 5 = 0$. Start with $x_1 = 1.5$. Then, in each case find x_2. 21) ______________

22) Use Newton's method to find the positive fourth root of 2 by solving the equation $x^4 - 2 = 0$. Start with $x_1 = 1$ and find x_2. 22) ______________

23) Marcus Tool and Die Company produces a specialized milling tool designed specifically for machining ceramic components. Each milling tool sells for \$4, so the company's revenue in dollars for x units sold is $R(x) = 4x$. The company's cost in dollars to produce x tools can be modeled as $C(x) = 301 + 29x^{5/8}$. Use Newton's method to find the break–even point for the company (that is, find x such that $C(x) = R(x)$). Use $x = 370$ as your initial guess and show all your work. 23) ______________

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find an antiderivative of the given function.

24) $\frac{7}{3}x^{5/3}$ 24) _______

A) $\frac{7}{8}x^{8/5}$ B) $\frac{7}{3}x^{8/3}$ C) $\frac{7}{5}x^{8/5}$ D) $\frac{7}{8}x^{8/3}$

25) $15x^2 + 6x + 4$ 25) _______

A) $6x^3 + 3x^2 + 4x$ B) $5x^3 + 3x^2 + 4x$ C) $5x^3 + 4x^2 + 4x$ D) $5x^3 + 3x^2 + 5x$

26) $5\sqrt{x} - 8$ 26) _______

A) $5x^{3/2} - 8x$ B) $\frac{10}{3}x^{3/2} - 8x$ C) $\frac{10}{3}x^{3/2} - 8$ D) $5x^{3/2} - 8$

27) $-\frac{24}{x^4}$ 27) _______

A) $-\frac{3}{x^8}$ B) $\frac{3}{x^9}$ C) $\frac{8}{x^3}$ D) $\frac{8}{x^4}$

28) $x^{-5} + \frac{1}{3\sqrt{x}}$ 28) ________

A) $-\frac{1}{4x^4} + \frac{2}{3}x^{1/2}$ B) $-\frac{1}{4x^5} + \frac{2}{3}x^{1/2}$ C) $-\frac{1}{5x^4} + \frac{2}{3}x^{1/2}$ D) $-\frac{1}{5x^5} + \frac{2}{3}x^{1/2}$

29) $x^9 - \frac{1}{x^9}$ 29) ________

A) $\frac{x^{10}}{10} - \frac{1}{10x^{10}}$ B) $\frac{x^{10}}{9} - \frac{1}{9x^8}$ C) $9x^8 + \frac{1}{9x^8}$ D) $\frac{x^{10}}{10} + \frac{1}{8x^8}$

30) $3 \cos 7x$ 30) ________

A) $3 \sin 7x$ B) $-21 \sin 7x$ C) $\sin 7x$ D) $\frac{3}{7} \sin 7x$

31) $\cos \pi x + 3 \sin \frac{x}{3}$ 31) ________

A) $-\sin \pi x - 9 \cos \frac{x}{3}$ B) $-\pi \sin \pi x + \cos \frac{x}{3}$

C) $\frac{1}{\pi} \sin \pi x - \cos \frac{x}{3}$ D) $\frac{1}{\pi} \sin \pi x - 9 \cos \frac{x}{3}$

32) $-\frac{4}{5} \csc^2 \frac{x}{5}$ 32) ________

A) $-4 \cot \frac{x}{5}$ B) $4 \cot \frac{x}{5}$

C) $-\frac{8}{5} \csc^2 \frac{x}{5} \cot \frac{x}{5}$ D) $-\frac{4}{25} \cot \frac{x}{5}$

33) $4 \csc 6x \cot 6x$ 33) ________

A) $-\frac{2}{3} \csc 6x$ B) $-\frac{2}{3} \cot 6x$ C) $-4 \csc 6x$ D) $\frac{2}{3} \csc 6x$

Find the most general antiderivative.

34) $\int \left(6t^2 + \frac{t}{9}\right) dt$ 34) ________

A) $18t^3 + \frac{2}{9}t^2 + C$ B) $12t + \frac{1}{9} + C$ C) $2t^3 + t + C$ D) $2t^3 + \frac{t^2}{18} + C$

35) $\int (2x^3 + 10x + 4)\, dx$ 35) ________

A) $6x^2 + 10 + C$ B) $2x^4 + 10x^2 + 4x + C$

C) $\frac{1}{2}x^4 + 5x^2 + 4x + C$ D) $6x^4 + 20x^2 + 4x + C$

36) $\int \left(\frac{1}{x^5} - x^5 - \frac{1}{5} \right) dx$ 36) _______

A) $-5x^4 - 5x^5 + C$

B) $\frac{1}{5x^6} - \frac{x^6}{6} - \frac{1}{5x} + C$

C) $\frac{-1}{4x^4} - \frac{x^6}{6} - \frac{x}{5} + C$

D) $\frac{1}{6x^6} - \frac{x^4}{4} + \frac{1}{25} + C$

37) $\int (\sqrt{t} - \sqrt[6]{t})\, dt$ 37) _______

A) $\frac{3}{2}t^{3/2} - \frac{7}{6}t^{7/6} + C$

B) $\frac{-1}{2}t^{1/2} - \frac{1}{6}t^{-5/6} + C$

C) $\frac{2}{3}t^{3/2} - \frac{6}{7}t^{7/6} + C$

D) $\sqrt{t} - \sqrt[5]{t} + C$

38) $\int \left(\frac{\sqrt{y}}{3} + \frac{5}{\sqrt{y}} \right) dy$ 38) _______

A) $\frac{1}{2}y^{3/2} + \frac{1}{10}\sqrt{y} + C$

B) $\frac{2}{9}y^{3/2} - 10\sqrt{y} + C$

C) $\frac{1}{6}\sqrt{y} - \frac{1}{10\sqrt{y}} + C$

D) $\frac{2}{9}y^{3/2} + 10\sqrt{y} + C$

39) $\int \frac{x\sqrt{x} + \sqrt{x}}{x^2}\, dx$ 39) _______

A) $-\frac{\sqrt{x}}{2} - \frac{3\sqrt{x}}{2} + C$

B) C

C) $\frac{2}{\sqrt{x}} - 2\sqrt{x} + C$

D) $2\sqrt{x} - \frac{2}{\sqrt{x}} + C$

40) $\int (-9 \cos t)\, dt$ 40) _______

A) $-\frac{9}{\sin t} + C$

B) $-\frac{\sin t}{9} + C$

C) $-9\sin t + C$

D) $-9 \cos t + C$

41) $\int (-4 \sec^2 x)\, dx$ 41) _______

A) $-4 \tan x + C$

B) $-4 \cot x + C$

C) $4 \cot x + C$

D) $\frac{\tan x}{4} + C$

42) $\int \sin \theta(\cot \theta + \csc \theta)\, d\theta$ 42) _______

A) $\cos \theta + C$

B) $\sin \theta + \theta + C$

C) $\sin \theta + C$

D) $\csc \theta + \cos \theta + C$

43) $\int \frac{\sec \theta}{\sec \theta - \cos \theta} d\theta$ 43) _______

A) $\theta + \tan \theta + C$ B) $\cot \theta + C$ C) $\cos^2 \theta + C$ D) $-\cot \theta + C$

Use differentiation to determine whether the integral formula is correct.

44) $\int (5x - 2)^4 dx = \frac{(5x - 2)^5}{25} + C$ 44) _______

A) Yes B) No

45) $\int \sec^2\left(\frac{x - 2}{2}\right) dx = -2 \cot\left(\frac{x - 2}{2}\right) + C$ 45) _______

A) Yes B) No

46) $\int x \sin x \, dx = -x \cos x + \sin x + C$ 46) _______

A) Yes B) No

Solve the problem.

47) Given the velocity and initial position of a body moving along a coordinate line at time t, find the body's position at time t. 47) _______
$v = -20t + 7$, $s(0) = 7$

A) $s = - 10t^2 + 7t - 7$ B) $s = - 10t^2 + 7t + 7$
C) $s = 10t^2 + 7t - 7$ D) $s = -20t^2 + 7t + 7$

48) Given the velocity and initial position of a body moving along a coordinate line at time t, find the body's position at time t. 48) _______
$v = \frac{8}{\pi} \sin \frac{4t}{\pi}$, $s(\pi^2) = 2$

A) $s = -2 \cos \frac{4t}{\pi} + 3.3073$ B) $s = -2 \cos \frac{4t}{\pi} + 4$

C) $s = -2 \cos \frac{4t}{\pi} + 8.2134$ D) $s = 2 \cos \frac{4t}{\pi} + 4$

49) Given the acceleration, initial velocity, and initial position of a body moving along a coordinate line at time t, find the body's position at time t. 49) _______
$a = 12 \cos 5t$, $v(0) = 8$, $s(0) = 8$

A) $s = \frac{12}{25} \sin 5t + 8t + 8$ B) $s = -\frac{12}{25} \cos 5t + 8t + 8$

C) $s = \frac{12}{25} \cos 5t - 8t + 8$ D) $s = -\frac{12}{25} \sin 5t + 8t + 8$

50) A rocket lifts off the surface of Earth with a constant acceleration of 30 m/sec^2. How fast will the rocket be going 2.5 minutes later? 50) _______

A) 37.5 m/sec B) 187.5 m/sec C) 75 m/sec D) −75 m/sec

1) C
2) B
3) A
4) As the trucker's average speed was 75 mph, the Mean Value Theorem implies that the trucker must have been going that speed at least once during the trip.
5) A
6) If x , y represent the legs of the triangle, then $x^2 + y^2 = 21^2$.

   Solving for y, $y = \sqrt{441 - x^2}$

   $A(x) = xy = x\sqrt{441 - x^2}$

   $A'(x) = -\dfrac{x^2}{2\sqrt{441 - x^2}} + \dfrac{\sqrt{441 - x^2}}{2}$

   Solving $A'(x) = 0$, $x = \pm\dfrac{21\sqrt{2}}{2}$

   Substitute and solve for y: $\left(\dfrac{21\sqrt{2}}{2}\right)^2 + y^2 = 441$; $y = \dfrac{21\sqrt{2}}{2}$ $\therefore x = y$.
7) A
8) C
9) D
10) C
11) B
12) B
13) B
14) A
15) B
16) A
17) C
18) A
19) $f(x) = 4x^2 + 3x - 2$, $f'(x) = 8x + 3$

   Right–hand solution:

   $x_1 = 0.5$

   $x_{n+1} = x_n - \dfrac{4x^2 + 3x - 2}{8x + 3} = \dfrac{4x^2 + 2}{8x + 3}$

   therefore $x_2 = 0.4286$

   Left–hand solution:

   $x_1 = -2$

   $x_{n+1} = x_n - \dfrac{4x^2 + 3x - 2}{8x + 3} = \dfrac{4x^2 + 2}{8x + 3}$

   therefore $x_2 = -1.3846$

20) $f(x) = -2x^3 - 3x - 4$, $f'(x) = -6(x^2) - 3$

$x_1 = -0.5$

$$x_{n+1} = x_n - \frac{-2x^3 - 3x - 4}{-6(x^2) - 3} = \frac{-4x^3 + 4}{-6x^2 - 3}$$

therefore $x_2 = -1.0000$

21) $f(x) = 2\sin x - 3x + 5$

$f'(x) = 2\cos x - 3$

$x_1 = 1.5$

$$x_{n+1} = \frac{2x\cos x - 2\sin x - 5}{2\cos x - 3}$$

therefore x_2 must be 2.37276989

22) $f(x) = x^4 - 2$, $f'(x) = 4x^3$

$x_1 = 1$

$$x_{n+1} = x_n - \frac{x^4 - 2}{4x^3} = \frac{3x^4 + 2}{4x^3}$$

therefore $x_2 = 1.2500$

23) Find the root of $f(x) = C(x) - R(x) = 301 + 29x^{5/8} - 4x$.

$$f'(x) = \frac{145}{8}x^{-3/8} - 4$$

$x_1 = 370$

$$x_2 = 370 - \frac{f(370)}{f'(370)} = 370 - \frac{301 + 29\cdot 370^{5/8} - 4(370)}{\frac{145}{8}\cdot 370^{-3/8} - 4} = 364.68$$

$$x_3 = 364.68 - \frac{f(364.68)}{f'(364.68)} = 364.68 - \frac{301 + 29\cdot 364.68^{5/8} - 4(364.68)}{\frac{145}{8}\cdot 364.68^{-3/8} - 4} = 364.67$$

The break-even point is $x = 364.67$ tools.

24) D
25) B
26) B
27) C
28) A
29) D
30) D

31) D
32) B
33) A
34) D
35) C
36) C
37) C
38) D
39) D
40) C
41) A
42) B
43) D
44) A
45) B
46) A
47) B
48) B
49) B
50) C